# MODULE 3 UNIT 4

## Optimising portfolio weights

### Table of contents

[Step 1 3](#_Toc113456019)

[1.1 Expected daily returns 3](#_Toc113456020)

[1.2 Variance and standard deviation 3](#_Toc113456021)

[Step 2 4](#_Toc113456022)

[2.1 Excess returns matrix *(Xc)* 4](#_Toc113456023)

[Step 3 5](#_Toc113456024)

[3.1 Covariance matrix 5](#_Toc113456025)

[Step 4 6](#_Toc113456026)

[4.1 Expected returns in an equally weighted portfolio 6](#_Toc113456027)

[4.2 Variance and standard deviation 6](#_Toc113456028)

[Step 5 7](#_Toc113456029)

[5.1 Finding the optimal portfolio weights 7](#_Toc113456030)

[5.2 Risk and return trade-off 8](#_Toc113456031)

Learning outcome:

LO4: Develop an investment portfolio associated with your risk appetite.

### Step 1

#### 1.1 Expected daily returns

The Excel spreadsheet associated with this workbook has a sheet named “Returns” that contains a set of simulated time series data representing the daily returns for five stocks: A, B, C, D and E.

Recall that daily returns are calculated as the percentage change in the closing price of an instrument from the previous day’s closing price. Therefore, it represents the gain or loss in the price of an instrument over a given period.

This workbook includes textboxes, such as the one below, notifying you when to navigate to the Excel spreadsheet to review certain features and functions.

Navigate to the Excel spreadsheet:

Review the “Returns” sheet containing the daily returns for stocks A, B, C, D, and E for each period.

Additionally, review the “Calculations” sheet containing a copy of the daily returns data in Columns A–F.

#### 1.2 Variance and standard deviation

You can calculate various statistics for each stock using the daily returns. For stock A, calculate the following:

* **Expected returns (Er):** Insert the formula “=AVERAGE(B4:B254)” with B4:B254 being the range that contains the daily returns for the stock.
* **Number of records (N):** Use the formula “=COUNT(B4:B254)”. Cell P5 is saved as a named range “N” for ease of reference in later calculations. You can do this by editing the name box in the top left corner of the rows and columns.
* **Sample variance:** Use the formula “=VAR.S(B4:B254)”.
* **Sample standard deviation:** Use the formula “=STDEV.S(B4:B254)”.

You can calculate the same statistics for Stocks B, C, D and E by copying and pasting these formulas in the relevant cells.

Navigate to the Excel spreadsheet:

Review the “Calculations” sheet containing the summary statistics. View cells P4–P7 to see the results of each variable for Stock A based on the formulas listed above. The formulas are copied and pasted for the rest of the range Q4:T7 to calculate the variables for the Stocks B, C, D and E.

Note:

To copy and paste a formula, you can select the relevant cell and drag the square across the cells you wish to paste the formula in.

### Step 2

#### 2.1 Excess returns matrix *(Xc)*

You can calculate the excess returns for a portfolio’s daily returns by subtracting the expected daily return for the stock from the observed daily return. The excess return matrix *(Xc)* is defined as the difference between a stock’s daily return (*X*) and its average return (*Er*). The formula for the excess return matrix is as follows:

*Xc* = *X* − *Er*

Navigate to the Excel spreadsheet:

Review the “Calculations” sheet containing the excess returns matrix in the cell range H4:L254. View Cell H4, which contains the formula “=B4-P$4”. This is used to calculate the difference between the daily return of Stock A (B4) and the expected daily return of Stock A (P4). This is repeated for all periods.

You add a dollar sign to cell reference P4 so that you can drag the formula across and below without changing the reference to Row 4. You want to keep the reference to Row 4 because the expected return is the same for each period. Therefore, as you calculate the excess returns for Stocks B, C, D and E across the entire range H4:L254, the reference always remains Row 4, with the column corresponding to the stock.

The formula is copied and pasted for every cell in the range H4:L254. Lastly, you can create a named range by selecting the excess return cells (H4:L254) and assigning it the name “Xc\_5s” for ease of reference in later calculations. Remember, once you’ve selected the cells, you can edit the name box in the top left-hand corner of the rows and columns.

### Step 3

#### 3.1 Covariance matrix

The covariance matrix is a statistical property that indicates how two or more variables correspond. Knowing the covariance between stocks helps to inform your diversification strategy, as you ideally want to include unrelated stocks or ones that move in opposite directions:

* If one variable moves in the same direction as a change in the other variable, the two variables have a **positive covariance**. Thus, two stock prices are likely to move in the same direction if they have a positive covariance.
* In contrast, if one variable moves in the opposite direction to another variable, the two variables have a **negative covariance**. Thus, a negative covariance indicates that the two stocks are likely to move in opposite directions.

The covariance relationship among a set of stocks can be mathematically described using a covariance matrix, also referred to as the variance-covariance matrix. The covariance matrix uses the excess returns (*Xc*) calculated in Step 2. The diagonal elements describe the variances of the variables, and the off-diagonal elements describe the covariances.

Navigate to the Excel spreadsheet:

Review the “Calculations” sheet containing the covariance matrix calculation in the cell range O10:T15.

Excel has the built-in functions “MMULT” for matrix multiplication and “TRANSPOSE” for transposing a matrix. View Cell P11, which contains the formula “=MMULT(TRANSPOSE(Xc\_5s),Xc\_5s)/(N-1)” used to calculate the covariance matrix for the range P11:T15.

“Xc\_5s” is the range referencing the excess returns matrix, previously named in Step 2. Also, “N” references the number of records in Cell P5, previously named in Step 1.

The covariance matrix also returns the same sample variance values calculated in Step 1 (P6:E6), which for this step are highlighted in blue.

Lastly, the covariance matrix in the cell range P11:T15 is assigned the name “covmat\_5s” for ease of reference in later calculations.

Note:

In some versions of Excel, you may need to enter an array formula. Instead of simply pressing enter, you need to simultaneously press the keys Cmd + Enter on Mac or Ctrl + Shift + Enter on Windows.

### Step 4

#### 4.1 Expected returns in an equally weighted portfolio

When a portfolio contains multiple stocks, you can calculate the expected return of the portfolio by multiplying the weight of each stock by its returns and adding them all together.

Navigate to the Excel spreadsheet:

Review the “Calculations” sheet containing the expected returns calculation for an equally weighted portfolio. You can use the function “SUMPRODUCT” to calculate the portfolio returns using the expected returns and weights of the individual stocks. Since the portfolio has equally weighted stocks, you can assume the weight is 20% in each stock for Cells P19:T19.

View Cell P20, which contains the calculation for the portfolio expected returns using the formula “=SUMPRODUCT (P19:T19; P4:T4)”. This uses the original expected returns for each stock calculated in Step 1 and the assumed equal weights of 20%.

#### 4.2 Variance and standard deviation

In addition to the expected returns of the equally weighted portfolio, you should evaluate the portfolio variance. The variance helps you determine the standard deviation and therefore the risk of the portfolio. This is calculated as a weighted variance using the covariance matrix.

Similar to Step 3, you can make use of the “TRANSPOSE” function and the built-in “MMULT” function to implement the weighted variance formula. Note that you need to perform the calculations in the right order to avoid incorrect results or errors.

Navigate to the Excel spreadsheet:

Review the “Calculations” sheet containing the weighted variance calculation for an equally weighted portfolio. View Cell P21, which contains the formula “=MMULT(MMULT(P19:T19,covmat\_5s),TRANSPOSE(P19:T19))” to calculate the portfolio variance.

Recall that the range of the covariance matrix was previously named “covmat\_5s” in Step 3.

Lastly, the standard deviation is calculated in Cell P22 by taking the square root of the variance in Cell P21 using the formula “=SQRT(P21)”.

### Step 5

The following steps to find the optimal portfolio weights is an optional exercise if you wish to find the portfolio that minimises the standard deviation and therefore the risk. Since you are minimising the risk, this step assumes that you are a risk-averse investor. Therefore, you can tailor the following steps to suit your own preferences depending on your risk appetite.

#### 5.1 Finding the optimal portfolio weights

An equally weighted portfolio should have a lower standard deviation than any of the individual stocks it contains. This is because you lower the risk by including more stocks in a portfolio. However, it is worth exploring if you can further reduce the risk by choosing a different set of weights.

You may be able to adjust the capital allocation towards less risky and safe stocks to reduce the overall portfolio risk. The goal is to update the values of the weights by minimising the standard deviation. Alternatively, you can minimise the variance since this will simultaneously minimise the standard deviation.

There are also two conditions that need to be met. These are that the weights need to add up to 100%, and their individual values must be positive.

Navigate to the Excel spreadsheet:

Review the “Calculations” sheet containing the calculation for optimal weights so that the standard deviation is minimised. Start by making a copy of the equally weighted portfolio scenario calculated in Step 4, ensuring the cell references are aligned to the correct data.

Then, use the Solver tool to find the optimal weights that meet a certain criterion, in this case, minimising the variance. The Solver tool can be accessed by navigating to the Data tab under the “Analyze” section.

If you cannot see the Solver tool in your version of Excel, please refer to your Excel version’s manual for instructions on activating the add-in as these instructions differ across Excel versions. Also note that the names of each feature in the Solver tool may differ across Excel versions. However, the steps remain the same.

Solver tool:

Once you have selected the Solver tool, select the cells you wish to optimise in the field “Set Objective”. In this case, the cell is P30 as you are optimising the standard deviation.

Select “Min” in the “To” field of the Solver parameters because you want to minimise the standard deviation.

In the “By Changing Variable Cells”, select the cells containing the weights you want to optimise. Therefore, you should input the weights in the cell range $P$27:$T$27, including the dollar signs to ensure references to these cells are maintained throughout.

In this case, there is no short-selling and all available funds must be invested. Therefore, you need to add a constraint that the weights must be positive and add up to 100%. You can do this by ensuring that the box “Make Unconstrained Variables Non-Negative” is checked. Also, you can add a constraint to ensure all variables sum to 100% by clicking on the “Add” button. The “Cell Reference” is Cell $U$27, whose value is the sum of the weights, and you can make this equal to a constraint value of 1.

Click “OK” once you are done to return to the main Solver tool dialogue. You will notice an entry of the defined constraint in the “Subject to the Constraints” section.

The “Select a Solving Method” can remain on its default of “GRG Nonlinear”.

Click on the “Solve” button to trigger the optimisation.

Results:

Once the Solver tool has been initiated with the constraints, it may take some time to run. Also, your Excel application may go black and white, especially if your application is running in the cloud. While the optimisation is running, a progress report may also appear on screen.

Once the Solver tool has completed the optimisation, it will indicate that a solution has been found and provide you with an option to either keep the solution or restore the original values. In this case, you want to keep the solution to determine what the optimal weights are.

Optimal weights:

The weights of the portfolio should be updated with the optimal values, shown in the blue cells P27:T27.

Additionally, the weighted standard deviation should have updated to the minimum value corresponding with the optimal portfolio weights. The original weighted standard deviation in the equally weighted portfolio was 0.864% in Cell P22. The updated weighted standard deviation in the optimised portfolio which minimises risk is 0.783% in Cell P30.

However, notice that the expected return (Er) has also decreased from −0.035% in Cell P20 to −0.042% in Cell P28.

#### 5.2 Risk and return trade-off

The objective of optimising the weights of each stock in the portfolio is to minimise the risk. This is achieved as the standard deviation decreases from the equally weighted portfolio. However, the expected return also decreases. Thus, this information is useful when making decisions about the risk and return trade-off. You should be guided by your risk appetite to determine whether you would prefer a portfolio with lower risk and lower expected returns, or with higher risk but higher expected returns.

After completing this workbook activity, consider whether you would invest in either the portfolio with equal or the one with optimal weights.